Te Wāhanga Tuaiwa ◊ Chapter Nine

Transformation geometry: Mā te nekehanga, mā te whakaata, mā te hurihanga

Hirāni Manuel, Ngārewa Hāwera and Merilyn Taylor

He pukepuke moana e ekengia e te waka.[1]

A choppy sea can be navigated.

He kōrero whakataki ◊ Introduction

The whakataukī (proverb) promotes the notion that navigating obstacles or barriers successfully is a natural part of learning. The learning and teaching of the Māori language and culture are key practices in Māori-medium settings. Teachers in such classrooms are actively promoting academic success as Māori through the medium of te reo Māori (the Māori language). This stance includes the learning of mathematics.

The ideas shared in this chapter emerged from a decision by the three authors to collaborate in the planning of a geometry unit to support Years 7 to 8 children at Te Kura Kaupapa Māori o Tōku Māpihi Maurea with their learning. The kura (school) is a Level 1 Māori-medium environment, where teaching and learning in te reo Māori are expected to occur 81–100% of the time.

This chapter shares three tasks out of six that were presented to children

1 http://maori.cl/Proverbs.htm

in an effort to offer them opportunities to explore panonitanga (transformation) geometry in various ways. The first author is an experienced teacher who has taught in Māori medium for more than 20 years. The other two authors are lecturers and researchers in mathematics education from a nearby university. From the outset the goal was to work collaboratively to share expertise and professional conversations so that children's mathematical learning could be optimised.

Learning mathematics in school can seem isolated from children's real-life experiences and thus inhibit their engagement and success (Sparrow, 2008). Attard (2012) suggests a number of factors that can help children to engage with mathematics. These include presenting children with tasks that:

- link to their world and prior knowledge
- allow for differentiation
- offer them choices
- provide them with opportunities to work collaboratively.

From the time that children begin their formal mathematical learning they are expected to have experiences that involve recognising and understanding geometrical transformation. Learning progressions in *Te Marautanga o Aotearoa* (Ministry of Education, 2008) indicate that by Year 7 children will have explored the translation, reflection, rotation and enlargement of various shapes. Children then need to develop further understanding of transformation by identifying, articulating and demonstrating their appreciation of properties such as line symmetry and rotational symmetry in two-dimensional and three-dimensional shapes or objects.

Te Marautanga o Aotearoa also promotes the use of digital technology to enhance learning experiences. Such technology is perceived to be an inherent part of everyday life and can encourage children to engage with tasks. Northcote (2011) states that when children take digital photographs, for example, the activity can support them to demonstrate their thinking and understanding of particular mathematical ideas.

Tasks for mathematics learning

Te Marautanga o Aotearoa and *The New Zealand Curriculum* documents (Ministry of Education, 2007, 2008) indicate that tasks should be presented to children within a range of meaningful contexts. The tasks that follow were used with children learning about transformation geometry.

1. Symmetry in the environment

Mā hea mai i tēnā ◊ Points to ponder

- How can we find out what ideas children currently hold about symmetry?
- How can we support children to link ideas of symmetry to their environment?
- What language do children use to describe symmetrical ideas?
- How can second-language acquisition techniques support connections between mathematical concepts and appropriate vocabulary?
- What home–school relationships can be fostered when exploring symmetry?

An important part of the teaching and learning process for mathematics is eliciting children's thinking about shape and space. Their environment can provide an easily accessible context for them to explore and explain their ideas about symmetry; for example, lines of symmetry, horizontal symmetry and vertical symmetry. Children can learn to appreciate that while a number of cultural objects in their society and those around the world are deemed inherently attractive in their own right, such objects might also exemplify symmetrical ideas.

Te ngohe ◊ Activity

Exploring symmetry in the environment

Show the children a PowerPoint of animals and other items in the environment (e.g., flowers, insects) and man-made artefacts (e.g., shoes, buildings) that illustrate symmetry. Questions posed to the children can include:

- Ina titiro koe ki ngā pikitia nei, he aha ōu kitenga?
  (What do you notice when you look at these pictures?)
- Ka kite tātou i ētehi mea pāngarau i roto i ngā pikitia?
  (Can we recognise any mathematics ideas relating to symmetry in the pictures?)

Have the children use digital media to take photographs of items in their environment that display symmetry.

Ask the children to download the photographs for a digital or hard-copy poster that can be displayed and discussed. Posters should illustrate and define symmetrical concepts. Software such as the Microsoft application Paint[2] can be used to show lines of symmetry.

2 http://windows.microsoft.com/en-NZ/windows7/products/features/paint

When she viewed the PowerPoint, the teacher remembered the valuable resource 'on her doorstep' that could be drawn on to contribute to her mathematics programme: "I like the idea of starting with the environment … Before I would've gone to find the Māori resources, gone online and looked at units." In the first conversation, when the partnership between the authors was established, the teacher had indicated that she was keen to explore alternative avenues to stimulate children's interest in this kaupapa (topic). She also wanted to refresh her content and pedagogical knowledge so that children could gain a broad perspective of geometric transformation beyond te ao Māori (the Māori world). The PowerPoint observation and digital photography task was an example to her of relevant activities that complemented the texts and online resources that had become her normal way of preparing for teaching.

The poster the children created from their photographs was a critical aspect of their learning and teacher assessment. Not all items that were photographed indicated 'true' symmetry. For example, upon closer scrutiny the fern frond in Figure 9.1 showed that each leaflet was not directly reflected on the opposite side of the stem. This observation gave the children and the teacher an opportunity to consider how the selection of each image might be justified.

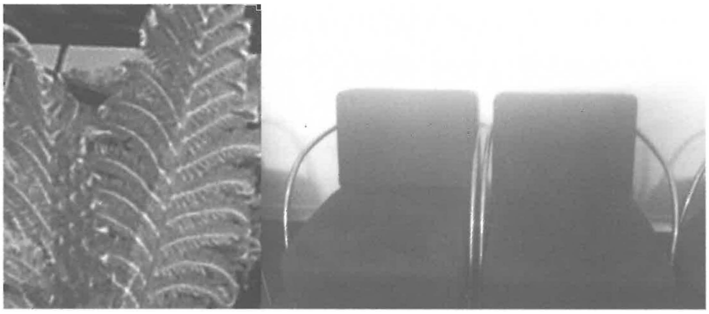


Figure 9.1: Children's photographs of objects in their environment perceived as symmetrical

The children needed to be able to articulate the mathematical reasons for selecting their items. Asking them to share their ideas in groups and then to the class gave them an opportunity to define the mathematics and verbalise their thinking. It also gave the teacher an opportunity to assess children's mathematical understanding and vocabulary. Following are some examples of statements made by children that illustrate their geometrical thinking.

Kei konei te tā moko. Kei waenganui te rārangi hangarite.

(Here is the tā moko. The line of symmetry is in the middle.)

Ina pōkai koe, ka ōrite. Ae, ka 'reopen' koe, ka hoki ki te tetipea tika.

(If you fold it, they are the same. Yes, when you 'reopen' it, it will go back to the right teddy bear.)

Ka noho te ringa 'exactly' ki runga i te ringa.

(The hand will sit exactly on the hand.)

Developing proficiency in te reo Māori and acquiring the necessary mathematical register can be challenging for children and teachers because of the specialist nature of the mathematical vocabulary (Meaney, Trinick, & Fairhall, 2012). Children's explanations for this task, however, indicated that they needed support with non-mathematical or supplementary words to communicate and justify their thinking. Their descriptions illustrated conceptual understanding of the mathematical ideas and showed that they could use (with a little reminding) key words such as hangarite (symmetry), rārangi poutū (vertical line), rārangi pae (horizontal line) and whakaata (reflection) appropriately (see the papakupu, glossary, at the end of this article for a full list of terms). Children needed to be reminded of or to learn alternative words to fully express their explanations with clarity in te reo Māori; e.g., tuwhera/huaki (open); noho marire (sit exactly), weheruatia (divided in half).

Supporting children to use known words or to extend their language while learning mathematics is a constant consideration in immersion contexts. The teacher's bilingual theory knowledge (e.g., Meaney et al., 2012; Gibbons, 2002) was important for organising and constructing explicit language acquisition tasks for children to complete in groups. For example, she presented as a cloze task the following sentence: "Ina weheruatia taua mea, ka taea tētehi wāhanga te whakanoho marire ki tētehi". (If the object is divided in half, one part will fit exactly onto the other) (Christensen, 2010, p. 106). The sentence was cut into individual words and the pieces mixed. The children had to construct a sentence using all of the words so that it made grammatical and mathematical sense of the transformation ideas they were being asked to communicate. The teacher routinely utilised second-language acquisition tasks such as this to ensure language and mathematical development could occur simultaneously and in context.

The whakataukī refers to navigating a choppy sea for learning. Deliberate planning and intent to support language acquisition with mathematical learning are essential in kura. Teacher knowledge of and expertise in second language acquisition in this case supported children to be successful in their mathematical learning and in their ability to express their understanding.

2. Exploring different patterns

Mā hea mai i tēnā ◊ Points to ponder

- What kinds of tasks can support children to develop a broad understanding of the significance of symmetry and geometrical transformation displayed by different cultural groups?
- How can we ensure that children have rich opportunities to identify and discuss these mathematical ideas?
- What links with te ao Māori can be made when teaching and learning mathematics?
- What are the issues in relation to using traditional Māori design when teaching and learning transformation geometry?
- What constitutes Western mathematics? What constitutes Māori mathematics?

Rich learning tasks can support children to learn mathematics. Such tasks can offer multiple pathways for learning and offer opportunities for children to speculate, hypothesise, discuss, make decisions and justify their responses (Ahmed, 1987). When considering transformation geometry, the identification of the mātāoroko (the original unit of a pattern that is translated) can incorporate key tenets of a rich task. The process of identification encourages exploration of symmetries and transformations that may occur within the units.

Understanding concepts of nekehanga (translation), hurihanga (rotation) and whakaata (reflection) within a range of simple to complex frieze patterns is important for recognising how such patterns are created. Offering children tracing paper to copy parts of patterns and then for practising reflecting, rotating and sliding items within repeated units can help them to recognise transformations and encourage the use of geometric language when describing their findings.

To assist children to investigate ideas of transformation across a variety of cultures, the authors constructed the following task.

Te ngohe ◊ Activity
Patterns and transformation

- Offer the children sheets with pictures of frieze patterns from various cultures around the world (e.g., Figure 9.2). Have them identify the mātāoroko in each pattern.
- Can they find and describe any transformations within the mātāoroko?

Te Marautanga o Aotearoa (Ministry of Education, 2008) and *Te Aho Matua* (Kura Kaupapa Māori Working Group, 1989) state that children at kura kaupapa Māori need to gain an appreciation of themselves, their own culture and their place in the world. Giving children a range of frieze patterns to investigate from their own and different cultures supports them to appreciate mathematical ideas inherent in indigenous cultural artefacts. Such tasks offer children an opportunity to broaden their views about the social application and importance of these concepts in the wider world.

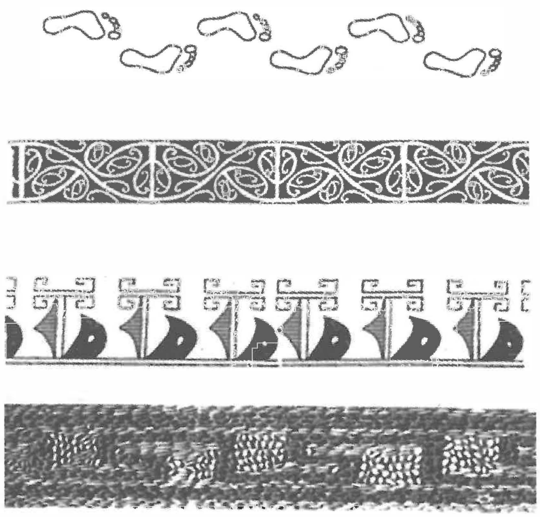


Figure 9.2: Examples of frieze patterns given to children

There are seven ways that an item can be transformed by translation, rotation, reflection, or a combination of these processes (see Appendix 9.1). The frieze patterns given to the children illustrated this full range of transformations. Some of the mātāoroko illustrated in the examples were easily discerned, while others required more thought. As well as encompassing the criteria of a 'rich' task, the activity aligned with the pedagogical intent of the teacher. Children were encouraged to work collaboratively and make choices about where to begin their exploration, and then set about highlighting the mātāoroko and any symmetries they could identify.

In this instance it was decided that after children had experience identifying mātāoroko and transformations in patterns, they could be offered opportunities to construct and practise transforming a physical item. They were asked to create a stylised stencil of an element from the natural world (e.g., a leaf or shell) to make their own mātāoroko for a frieze pattern. However, some children chose small two-dimensional items (e.g., a plastic fish) instead from their mathematical equipment to manipulate because they found these easier to manoeuvre. Eventually, children need to be able to imagine such manipulation and the consequences of it in practice, which will help enable them to work with computer technologies, which offer greater power and sophistication for transforming shape (Bobis, Mulligan, & Lowrie, 2009).

Some examples of geometric transformation are quite complex and challenging to identify. In this instance, the authors found it necessary to clarify the transformation ideas embedded in the patterns that were to be presented to the children. Wānanga (conversations) focused on content ideas and language issues. Were we clear ourselves about the transformations in the patterns? Where is the centre of rotation that is used to create the pattern? How does the word 'pūtake' (unit being transformed) fit into this idea? Should we be promoting the word 'pūtake' instead of the term 'mātāoroko', which is suggested in the Christensen (2010) pāngarau dictionary? Are we going to broaden children's vocabulary or just confuse their understanding of the mathematics if we use different words for the same idea?

These questions (and more) contributed to wānanga that occurred between the authors, and illustrate the complexity of working in this topic. The practice of wānanga between teacher and researchers links to the whakataukī, because it facilitated greater clarity and confidence in teaching the mathematical ideas and the requisite reo Māori. While the process of wānanga takes time and commitment, it does mean that the 'sea' becomes more navigable and progress can be made.

3. Visiting a familiar setting

Mā hea mai i tēnā ◊ Points to ponder

- What other contexts can we use to maintain children's engagement with symmetry and transformation?
- What sites in the local community are readily available to support and enhance the learning of transformation geometry?
- How do we help children to view and appreciate familiar and traditional artefacts through a mathematical lens?

For some children in Aotearoa, a visit to their local marae (traditional building complex of Māori) might be a relatively new experience. Designating any part of a marae complex as a site for mathematical learning must be carefully planned and facilitated. Learners need opportunities to gain an understanding of the various aspects of a marae and to appreciate its significance to local Māori. A study of any particular component of a marae has to be managed so that the integrity of the marae and what it signifies to its local community is kept to the fore.

For children in a kura kaupapa Māori, the marae is a meaningful site that is a familiar social, cultural and spiritual gathering place. Children come to understand the relevance and importance of this context for a local Māori community. While there is a risk that some Māori children might be overexposed to such settings, the authors learned that exploring aspects of traditional contexts through an alternative lens can support learners' mathematical understanding.

Te Ngohe ◊ Activity

Community and transformation geometry connections

- Take the children to the local marae.
- Ask a kaumātua (elder) to explain the stories behind the kōwhaiwhai (painted rafter) patterns.
- Ask the children to sketch or take photographs of the kōwhaiwhai panels, then identify and label the mātāoroko and any transformations they notice.

The kōwhaiwhai patterns and carvings portrayed on a marae complex depict unique historical sites, people and cultural events related to that particular hapū (subtribe) and provide opportunities for exploring geometric transformation. For the children and teacher this was their first time they critically examined familiar patterns through mathematical 'eyes'. As the teacher said, "[it was] good to go to the marae and focus on the maths ... [we] could 'see' the maths too when we visited another marae later."

Community involvement is a crucial component of kura kaupapa Māori philosophy, whereby whānau (family) are considered to be an integral part of the learning process. At the marae, a kaumātua made himself available to explain the cultural significance illustrated by the various designs in the wharenui (meeting house). This marae visit is an example of organising opportunities for others to share their 'expert' mātauranga (knowledge), aligned with the teacher's intent to demonstrate that knowledge can come from various sources, and the knowledge from various sources can be woven together.

The kōrero (talk) by the kaumātua helped the children, teacher and researchers to understand the origin of the kōwhaiwhai patterns and what they represented. An example of a kōwhaiwhai panel in the wharenui is given in Figure 9.3. Children learned that this pattern depicts the main awa (river) that links to the Tainui whakataukī "He piko he taniwha. He piko he taniwha" (At every bend there is a chief). The maunga (mountain) for this iwi (tribe) is illustrated by the smaller peaks surrounded by curved white lines, symbolising the sorrow felt for those tūpuna (ancestors) buried on the mountain. The children were able to clearly distinguish the mātāoroko that was being repeated and identify a vertical line of symmetry in the centre of the unit.

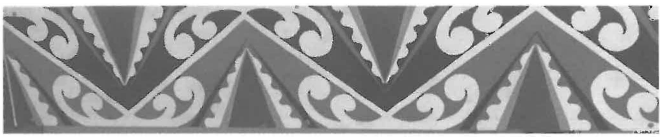


Figure 9.3: Rafter pattern, Te Ao Hurihuri, University of Waikato

Children come to understand that the patterns on a marae have meaning for the local community and can display elegant and complex mathematical ideas. Items are not constructed in isolation, but are the result of careful consideration of significant people, places and events. These children learned how tribal stories, reo Māori and transformation geometry were all connected and could be viewed in an integrated, meaningful way. Schools and teachers who wish to avail themselves of this context for mathematical learning would need to ensure they consider the significance of the whole marae complex and the importance of each component within it.

The teacher and researchers appreciated the role the kaumātua played in this task and the mātauranga he brought to it. His background knowledge of the social and cultural significance of the kōwhaiwhai patterns ensured greater 'richness' for children's learning. The whakataukī at the beginning of this chapter urges teachers and whānau to explore all avenues to make sure that children's experiences in mathematics education are rich and designed to optimise opportunities for success.

Whakarāpopoto ◊ Summary

Tātaiako: Cultural Competencies for Teachers of Māori Learners challenges teachers to seek ways of connecting with "the vibrant contemporary Māori values and norms" (Ministry of Education, 2011, p. 3) that are part of the cultural heritage of New Zealand. Although this chapter is about teaching transformation geometry in a Māori-medium setting, we believe that the tasks described here are relevant to all learners.

Listening to children while they explored the tasks provided the teacher with opportunities to formatively assess their mathematical learning and associated language development. As a consequence, the teacher was in a stronger position to create or modify tasks to meet the specific needs of the children. The benefits for all participants from this experience are linked to the fundamental concept of ako (reciprocal learning). Progress was made by the children with their learning of transformation geometry using a range of media. Further insights were gained by the teacher for extending and broadening her planning and pedagogical practice, and researchers developed a greater appreciation of the challenge, delight and privilege of teaching and learning mathematics in this particular kura kaupapa Māori.

He mihi

E mihi ake ana mātou ki te whānau o Te Kura Kaupapa Māori o Tōku Māpihi Maurea i whakaae kia mahitahi i te kaupapa nei. Ko te tūmanako ka whai hua ēnei mahi hei oranga mō ā tātou tamariki i roto i ngā kura katoa.

Te ngohe ◊ Activity

Have the children construct their pattern on a computer for an e-portfolio.

Ask the children to explain their frieze patterns to parents at a parent-teacher conference.

Have children construct a frieze pattern for:

- a waka āma T-shirt, or
- a carry-on bag for members of a New Zealand team going to an international event.

Ngā rauemi whai ake ◊ Follow-up reading

Hāwera, N., & Taylor, M. (2013). Children's views about geometry tasks in Māori-medium schools: Meeting Ngā Whanaketanga Rumaki Māori pāngarau (National Standards in mathematics). *set: Research Information for Teachers, 3*, 37–46.

Jorgensen, R., & Dole, S. (2011). *Teaching mathematics in primary schools* (2nd ed.). Crows Nest, NSW: Allen & Unwin.

McMurchy-Pilkington, C., Trinick, T., & Meaney, T. (2013). Mathematics curriculum development and indigenous language revitalization. *Mathematics Education Research Journal, 25*(3), 341–360.

Meaney, T., Fairhall, U., & Trinick, T. (2007). Acquiring the mathematics register. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice: Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia, Hobart, Tasmania, 2–6 July* (Vol. 2, pp. 493–502). Adelaide, South Australia, MERGA.

Snape, T. (2010). What is HOT about working as a mathematician?: Keeping the interest and challenge high for all students. In R. Averill & R. Harvey (Eds.), *Teaching primary school mathematics based statistics: Evidence based practice* (pp. 181–195). Wellington: NZCER Press.

See the Addendum for further uses of digital technology for mathematics learning.

Ngā huanga tautoko ◊ References

Ahmed, A. (1987). *Better mathematics: A curriculum development study.* London, UK: Her Majesty's Stationery Office.

Attard, C. (2012). Engagement with mathematics: What does it mean and what does it look like? *Australian Primary Mathematics Classroom, 17*(1), 9–12.

Bobis, J., Mulligan, J., & Lowrie, T. (2009). *Mathematics for children: Challenging children to think mathematically*. Frenchs Forest, NSW: Pearson Education Australia.

Christensen, I. (2010). *Te reo pāngarau: A Māori language dictionary of mathematics.* Wellington: Ministry of Education.

Gibbons, P. (2002). *Scaffolding language scaffolding learning: Teaching second language learners in the mainstream*. Portsmouth, NH: Heinemann.

Kura Kaupapa Māori Working Group. (1989). *Te aho matua*. Wellington: Ministry of Education.

Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in indigenous mathematics classrooms*. New York, NY: Springer.

Ministry of Education. (2007). *The New Zealand curriculum.* Wellington: Learning Media.

Ministry of Education. (2008). *Te marautanga o Aotearoa*. Wellington: Author.

Ministry of Education. (2011). *Tātaiako: Cultural competencies for teachers of Māori learners.* Wellington: Author.

Northcote, M. (2011). Teaching with technology. *Australian Primary Mathematics Classroom*, *16*(3), 29–32.

Sparrow, L. (2008). Real and relevant mathematics: Is it realistic in the classroom? *Australian Primary Mathematics Classroom*, *13*(2), 4–8.

Te papakupu ◊ Glossary

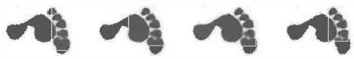
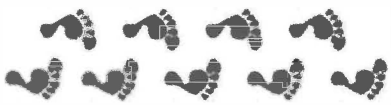
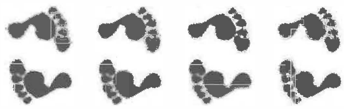
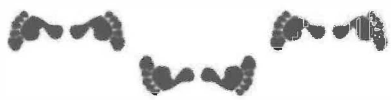
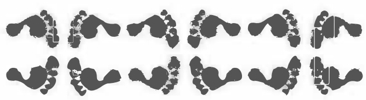
hangarite	symmetry
hurihanga	rotation
kaumātua	elder
kaupapa	topic
kōwhaiwhai	painted rafter pattern
kura	school
marae	traditional building complex of Māori
mātāoroko	the original unit of a pattern that is translated
mātauranga	knowledge
mātauranga Māori	Māori knowledge
nekehanga	translation
te ao Māori	the Māori world
te reo Māori	the Māori language
wānanga	conversation/ discussions

whakaata	reflection
whakataukī	proverb
wharenui	meeting house

Ngā pūkōrero ◊ Author information

Hirāni Manuel (Ngāti Tūwharetoa, Ngāti Raukawa, Ngāti Maniapoto) has been a teacher in Māori medium for more than 20 years. She was at Te Kura Kaupapa Māori o Tōku Māpihi Maurea and has recently become a Resource Teacher of Māori based in Tūwharetoa. Ngārewa Hāwera (Tainui) and Merilyn Taylor work in initial teacher education at Te Kura Toi Tangata (Faculty of Education), University of Waikato. They share an interest in mathematics education for Māori learners.

Appendix 9.1: Transformations

Frieze symmetry bingo	
Hop (Te hītoko) • Translation symmetry — the pattern is unchanged if you slide the pattern along.	
Step (Te hīkoi) • Translation • Glide reflection—the pattern is unchanged if you slide it along and reflect it in a horizontal line.	
Sidle (Te nekeneke) • Translation • Vertical reflection—the pattern is unchanged if you reflect it in a vertical line.	
Spinning hop (Te hītoko hurihuri) • Translation • Rotation—the pattern is unchanged if you spin it by a half turn.	
Spinning sidle (Te neke hurihuri) • Translation • Glide reflection • Rotation	
Jump (Te pekepeke) • Translation • Horizontal reflection	
Spinning jump (Te peke hurihuri) • Translation • Horizontal and vertical reflection • Rotation	

Available at https://docs.google.com/a/waikato.ac.nz/document/d/1vf_D5AEC1chj3EWuvbLga7N9IywTDDwKWgBvSCISwFs/edit?hl=en_GB#heading=h.oh09rdwkdcbv)